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DEVELOPMENT OF QUASIHARMONIC MOTIONS OF A GAS-STREAMLINED LIQUID FILM

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UDC 532.62+532.592+594

Quasiharmonic wave motions of a thin liquid film flowing in a vertical plane due to gravitational force, capillary forces, and a tangential stress acting on the film–gas boundary are considered. The region of existence and spectral characteristics of the quasiharmonic wave solutions in different film-motion regimes (cocurrent and countercurrent) are found.

§1. Let us consider the motion of a thin film of a viscous liquid flowing in a vertical plane, under the influence of gravitational and capillary forces and of stresses arising on the film surface as it is streamlined by gas. As in [1, 2], we replace the closed combined motion problem of the gas and liquid (in the film) by motion problems of a single film only. The effect of the gas on the film in the problem thus reduced is described by specifying the tangential (and normal) stresses on the gas–film boundary. The exact form of these stresses is unknown within the context of this procedure. We assume, as in [1, 2], that the tangential stress on the film

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surface is constant throughout the length of the film (this is true if the wave amplitude on the film surface is sufficiently low).

Thus the tangential stress will occur in the problem as a given parameter, and an estimate of its magnitude under actual conditions can be carried out by means of known (semiempirical) formulas relating the tangential stress to gas velocity (cf. for example, [3]).

In this formulation, film motion is described by the equations

$$\frac{\partial \tilde{v}_i}{\partial \tilde{t}} + \tilde{v}_k \frac{\partial \tilde{v}_i}{\partial \tilde{x}_k} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial \tilde{x}_i} + \nu \Delta \tilde{v}_i + \tilde{g}_i; \quad \frac{\partial \tilde{v}_i}{\partial \tilde{x}_i} = 0$$

with boundary conditions

$$\tilde{v}_i = 0 \text{ when } \tilde{y} = 0;$$

$$\tilde{v}_y(\tilde{h}) = \frac{\partial \tilde{h}}{\partial \tilde{t}} + \tilde{v}_x(\tilde{h})\frac{\partial \tilde{h}}{\partial \tilde{x}},$$

$$n_k = \left[-\tilde{p}\delta_{ik} + \rho\nu\left(\frac{\partial \tilde{v}_i}{\partial \tilde{x}_k} + \frac{\partial \tilde{v}_k}{\partial \tilde{x}_i}\right)\right] - n_k \tilde{T}_{ik} = \frac{\sigma}{\tilde{R}} n_i,$$

where $\tilde{y} = \tilde{h}(\tilde{x}, \tilde{t})$. Here ρ and ν are liquid density and viscosity; σ is the coefficient of surface tension of the liquid at the boundary with the gas, g is the acceleration of gravity, $\tilde{T}_{ik}$ is the stress tensor on the film–gas boundary, and R is the radius of curvature of the film surface. The coordinates are selected such that the film occupies the region $0 < \tilde{y} \leq \tilde{h}(\tilde{x}, \tilde{t})$, $-\infty < \tilde{x} < \infty$ ($\tilde{x}$ is coordinate); the gas occupies the region $\tilde{y} > \tilde{h}(\tilde{x}, \tilde{t})$; dimensional variables are given the index ~.

We select steady-state motions in the form of long plane waves from all possible motions of the film. For such waves, the problem reduces to the equation [1, 2, 4]

$$\frac{d^3h}{d\tau^3} + f(h)\frac{dh}{d\tau} = \varepsilon g(h), \quad f(h) = \frac{6q^2}{5h^3} - \frac{c^2}{5h} - \frac{3cT}{20} + \frac{3qT}{40h} - \frac{9T^2h}{160}, \tag{1.1}$$

$$g(h) = -1 + \frac{c}{h^2} - \frac{q}{h^3} - \frac{3T}{4h}$$

with the additional condition

$$\langle h \rangle = 1, \text{ where } \langle \ldots \rangle = \lim_{|\tau_2 - \tau_1| \to \infty} \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} (\ldots)\, d\tau \tag{1.2}$$

averaged over the length of the film.

Here h, c, (−q), and T are properly dimensionless variables of film thickness, "phase" wave velocity, flow rate (in a frame bound to the moving wave), and the tangential stress on the surface. Our notation are related to the data of the problem by the equations

$$\tau = W(x - ct); \quad \varepsilon = \frac{27\nu^4}{g_+^2 \langle \tilde{h} \rangle^{11/2}} \left(\frac{\sigma}{\rho}\right)^{1/2},$$

where $1/W^2 = 9\nu^2\sigma/\rho g_+^2 \langle \tilde{h} \rangle^5$ is the Weber number, $g_+ = g - (1/\rho)(d\tilde{p}_+/d\tilde{x})$, and p_+ is gas pressure. Mean film thickness $\langle \tilde{h} \rangle$ is selected as the scale for measuring lengths, $g_+ \langle \tilde{h} \rangle^2 / 3\nu$ is selected for measuring velocities, and $\frac{1}{2}\rho g_+ \langle \tilde{h} \rangle$ for measuring stresses.

Equation (1.1) differs from the corresponding equation from [1, 2], in that other scales and dimensionless parameters were selected (tangential stress on film surface was referred to gravitational force and was positive as gas moved downward and negative as it moved upward).

§2. To find the periodic solution of the problem (1.1), (1.2) we apply a method used for calculating wave motions of a freely draining film [5]. We first integrate Eq. (1.1) over τ from some fixed value τ_0 to τ. Since the term $f(h)(dh/d\tau)$ is a total derivative and can therefore be written in the form $f(h)(dh/d\tau) = (d/d\tau)[dV(h)/dh]$, we obtain

$$\frac{d^2h}{d\tau^2} + \frac{\partial V(h)}{\partial h} = \varepsilon F(h, \tau), \tag{2.1}$$

where

$$F(h, \tau) = \int_{\tau_0}^{\tau} g(h)\, d\tau.$$

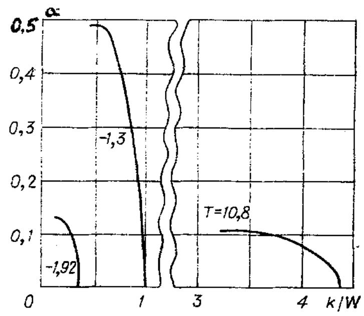

Fig. 1

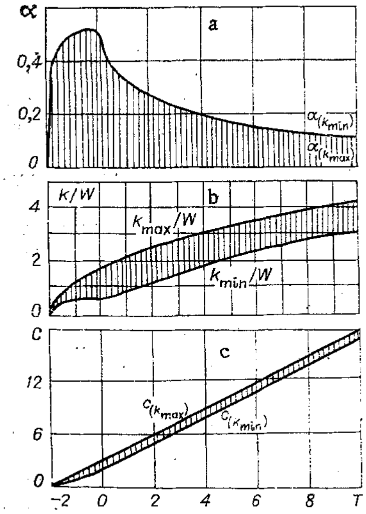

Fig. 2

Equation (2.1) can be interpreted as an equation that describes mechanical motion occurring due to the nonlinear potential force $\partial V/dH$ and nonpotential force $F(h, \tau)$. In this interpretation, the set of periodic motions found is simply the set of oscillatory motions occurring in a "potential pit" $V(h)$ and subject to the effect of a "nonpotential force" $F(h, \tau)$.

It is clear from physical concepts that the profile of these "oscillations" when $\varepsilon \ll 1$ will be completely determined by the form only of the potential pit, while the role of the nonpotential force $F(h, \tau)$ becomes understandable if we note that the nonpotential force $F(h, \tau)$ will not typically be completed during an oscillation period if strictly periodic solutions (limit cycles) are found.

Thus we may formulate the method of constructing the periodic solutions of the problem (1.1), (1.2) when $\varepsilon \ll 1$, which, on the one hand, defines the set of all oscillatory solutions in the potential pit $V(h)$, and, on the other hand, eliminates those for which the "force effort" $F(h, \tau)$ periodically is nonzero and thus will finally turn out to be nonperiodic. Since the "mean force effort" F is defined by $\langle (dh/d\tau)F\rangle$, using Eq. (2.1) we obtain an equation defining the rule for discarding limit cycles:

$$\left\langle \frac{dh}{d\tau} F \right\rangle = h(\tau)\langle g(h)\rangle - \langle hg(h)\rangle = 0. \tag{2.2}$$

Since $\langle g\rangle$ and $\langle hg\rangle$ are constants while $h(\tau)$ is a variable, condition (2.2) will hold if each of the terms in the left side of Eq. (2.2) independently vanishes; i.e., condition (2.2) is equivalent to the two conditions

$$0 = \langle g(h)\rangle = -1 + c\langle h^{-2}\rangle - q\langle h^{-3}\rangle - \frac{3T}{4}\langle h^{-1}\rangle; \tag{2.3}$$

$$0 = \langle hg(h)\rangle = -1 + c\langle h^{-1}\rangle - q\langle h^{-2}\rangle - \frac{3T}{4}. \tag{2.4}$$

Conditions (2.3) and (2.4) can also be obtained directly from Eq. (1.1). In fact, if we assume that the solution $h(\tau)$ in Eq. (1.1) is bounded and does not vanish (the film does not adhere to the wall), we obtain Eq. (2.3) by averaging Eq. (1.1) with respect to the variable τ, taking into account the fact that total derivatives occur on the left in Eq. (1.1). We obtain Eq. (2.4) by carrying out the same averaging procedure after first multiplying Eq. (1.1) by $h(\tau)$, noting that $h(d^3h/d\tau^3)$ and $hf(h)(dh/d\tau)$ remain total derivatives.

Let us give a concrete physical meaning, beyond the interpretation above, to the integral equations (2.3) and (2.4). It is clear from studying the occurrence of the terms in the function $g(h)$ in Eq. (1.1) in deriving this equation from the Navier–Stokes equations and the boundary conditions that $g(h)$ is (within a constant factor) the sum of the gravitational force and frictional force on the wall, and the interaction force with the gas acting on a unit thickness of the film cross section. This makes clear the origin of the rule of discarding the limit cycles (2.3) and (2.4), which shows that forces acting on the film in steady-state motion will be balanced on the average throughout the length of the film.

Let us consider the construction of periodic solutions of the initial problem and the case $\varepsilon \ll 1$ which is of most interest for applications. The solution of the problem in this case will be in the form of an asymptotic decomposition with respect to the small parameter ε,

$$h(\tau)=\sum_{n=0}^{\infty}\varepsilon^n h_n(\tau).$$

We limit ourselves to a first approximation and give h_0 the previous meaning for $h(\tau)$, leaving us in this approximation with

$$\frac{d^3h}{d\tau^3}+f(h)\frac{dh}{d\tau}=0. \tag{2.5}$$

Equations (2.5) remains nonlinear, and the nature of the decomposition of conditions (2.3) and (2.4) does not impose a priori constraints on the amplitude.

Equation (2.5) for given T has a five-parameter family of solutions (three constant integrations and the parameters q and c). It is necessary to eliminate from this family only those constrained periodic solutions that satisfy conditions (2.3) and (2.4). Moreover, the solution of the problem must satisfy condition (1.2) as well as a condition following from the arbitrariness of selecting the origin on the τ axis (initial wave phase). These conditions distinguish a one-parameter (for every value of T) set of wave solutions from the five-parameter family.

§3. We obtain the two equations

$$-1+c-q-\frac{3}{4}T+0(\psi^2)=0;$$

$$\left(-2c+3q+\frac{3}{4}T\right)\langle\varphi^2\rangle+0(\varphi^4)=0\,,$$

for low-amplitude solutions, representing $h(\tau)$ in the form $h=1+\varphi(\tau)$, $\langle\varphi\rangle=0$ [thereby already satisfying condition (1.2)], and satisfying the remaining two conditions (2.3) and (2.4).

We find $c=3[1+(T/2)]+0(\varphi^2)$, and $q=2+{}^3\!/\!_4T+0(\varphi^2)$ by solving these equations for c and q. For the wave number we have

$$k^2=\frac{6}{5}q^2-\frac{c^2}{5}-\frac{3cT}{20}+\frac{3qT}{40}-\frac{9T^2}{160}+0(\varphi^2)=3\left(1+\frac{T}{2}\right)+0(\varphi^2)\,,$$

after substituting these equations in Eq. (2.4). When $T<-2$, this equation leads to imaginary values of the wave number, so that the periodic quasiharmonic solutions do not contain low-amplitude solutions (as will be proved below, all quasiharmonic stationary waves also vanish).

We integrate Eq. (2.5) over τ twice in accordance with the method of constructing the periodic solutions. We obtain

$$\frac{1}{2}\left(\frac{dh}{d\tau}\right)^2+V(h)=0,\quad V(h)=\frac{3q^2}{5h}+\left(\frac{3qT}{40}-\frac{c^2}{5}\right)(h\ln h-h)-\frac{3cT}{40}h^2-\frac{3T^2h^3}{320}+A_1h+A_2. \tag{3.1}$$

In accordance with the interpretation given above, V(h) plays the role of a potential and $\frac{1}{2}(dh/d\tau)^2$, that of kinetic energy.

It is convenient to introduce the minimal and maximal film thicknesses in the wave profile $h(\tau)$ in place of the constants of integration A_1 and A_2. Denoting them by h_1 and h_2, we have $V(h_1)=V(h_2)=0$. The equation for V(h) then takes the form

$$V(h)=\Phi(h)-\frac{\Phi(h_1)(h_2-h)+\Phi(h_2)(h-h_1)}{h_2-h_1},$$

where

$$\Phi(h)=\frac{3q^2}{5h}+\left(\frac{3qT}{40}-\frac{c^2}{5}\right)(h\ln h-h)-\frac{3cT}{40}h^2-\frac{3T^2h^3}{320}.$$

We determine $\langle h^m\rangle$ from Eq. (4.1) using the equation

$$\langle h^m\rangle=\frac{1}{\lambda}\int_0^\lambda h^m(\tau)\,d\tau=\frac{I_m}{I_0},\ \text{where}\ I_m=\int_{h_1}^{h_2}\frac{h^m dh}{\sqrt{-V(h)}}, \tag{3.2}$$

and substitute $\langle h^m\rangle$ in Eqs. (1.2), (2.3), and (2.4), obtaining the system of equations

$$I_1 = I_0; \quad c = \frac{I_0(I_{-2} - I_{-3}) + \frac{3}{4} T (I_{-1} I_{-2} - I_0 I_{-3})}{I_{-2}^2 - I_{-1} I_{-3}}; \tag{3.3}$$

$$q = \frac{I_0(I_{-1} - I_{-2}) + \frac{3}{4} T (I_{-1}^2 - I_0 I_{-2})}{I_{-2}^2 - I_{-1} I_{-3}}; \quad k = \frac{\pi W \sqrt{2}}{I_0},$$

to determine h_1, h_2, c, q, and the dimensionless wave number k; this system of equations was solved on a computer. The solution can be theoretically carried out by specifying a set c, q, h_1, and h_2 for fixed T, after which the integrals I_m are calculated using Eqs. (3.2). The set c, q, h_1, and h_2 varies until it is self-consistent, i.e., until the system (3.3) becomes an identity. The collection of such self-consistent sets turns out to be a one-parameter collection for every value of T.

Such an order of computation would be highly cumbersome because of the large number of variables occurring in the set. Therefore, transformations sharply shortening the volume of computations were first carried out. These transformations, though differing for different valuation domains of T, are of the same type and, as a result, we will limit ourselves to indicating their form only in the region $T > 0$. We introduce the new variables $\theta = (c^{3/2}/\sqrt{5q})\tau$ and $H = (c/q)h$ and let $u = 3qT/8c^2$. Then Eq. (1.1) has the form

$$\frac{d^3 H}{d\theta^3} + \frac{dH}{d\theta}\left(\frac{12}{H^3} - \frac{1 + 2u^2}{H} + 2u - H\right) = 0, \tag{3.4}$$

i.e., it contains only the single parameter u in place of the three parameters c, q, and T, where u corresponds to the parameter T. The unknown parameters are now only the variables H_1 and H_2, the minimal and maximal film thicknesses in the wave profile $H(\theta)$. Eqs. (1.2), (2.3), and (2.4) can be combined into a single condition and containing only u and $\langle H^m \rangle$ as variables connecting H_1 and H_2;

$$2u - \langle H^{-1} \rangle + \langle H^{-2} \rangle = \langle H \rangle [2u \langle H^{-1} \rangle - \langle H^{-2} \rangle + \langle H^{-3} \rangle]. \tag{3.5}$$

The last two additional conditions (together with the equation $u = 3qT/8c^2$) allow us to find c, q, and T after solving the problem (3.4), (3.5). The order of the computations is defined as follows. A set (H_1, H_2) is specified for a fixed value of u and the integrals $K_m = \int_{H_1}^{H_2} \frac{H^m dH}{\sqrt{-V(H)}}$. are calculated. The set (H_1, H_2) varies until the self-consistency condition (3.5) is satisfied. The collection of such self-consistency sets (H_1, H_2) for fixed u constitutes a one-parameter family, that is, a curve on the (H_1, H_2)-plane.

Results of the calculations are depicted in Figs. 1-3. Figure 1 represents typical dependences of the dimensionless amplitude on the dimensionless wave number for three values of T. Clearly, only a comparatively narrow spectrum of the wave numbers corresponding to stationary quasiharmonic wave motions of the film exists for each value of the tangential stress. The hatched regions in Figs. 2 and 3 include precisely the waves from this spectrum. Figures 2a, b, and c demonstrate the evolution of the spectral wave characteristics (amplitude, wave numbers, and phase velocity) as the tangential stress varies. Theoretical dependences of wave velocities on gas velocity under cocurrent and countercurrent conditions are compared in Fig. 3 to experimental data taken from [6]. Tangential stress (and Reynolds number Re) is converted to gas velocity in the tube according to the equations

$$\tilde{T} = \frac{\lambda_\alpha}{8} \rho_2 \tilde{v}_2^2, \quad \lambda_\alpha = 0.11 \left(\frac{68}{\mathrm{Re}} - \frac{2\tilde{\alpha}}{d}\right)^{1/4},$$

where $\tilde{v}_2$ is the mean gas velocity in the tube, ρ_2 is its density, λ_α is the coefficient of friction in a tube with irregularities (the equation for λ_α was taken from [7]), α is wave amplitude, and d is tube diameter.

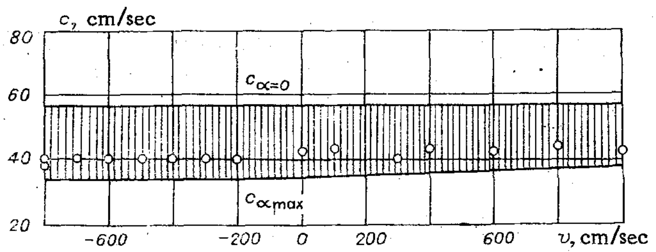

Fig. 3

It is assumed that waves play the role of irregularities here, since, as in [1], the dimension of the irregularities in the equation for λ_α is replaced by half the amplitude.

In conclusion, let us indicate the boundaries within which the theoretical conclusions hold. They are determined by the requirement that the wave number k [$k \ll 1$, which is the basis for deriving Eq. (1.1)], the parameter ε ($\varepsilon \ll 1$, which is the basis for constructing the solution), and the amplitude are all small, allowing us to disregard the dependence of tangential stress on wave profile. By imposing these requirements on ε, k, and α, we find that these requirements hold when Re is less than 10^4-10^5 if $10 < Re < 100$ in the case of water, for example, under downstream cocurrent conditions.

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DROP EVAPORATION IN A TURBULENT GAS JET

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UDC 532.517.4

Evaporation of a semidispersive drop system in a turbulent gas jet is considered. A method for calculating drop evaporation in a turbulent gas jet is proposed based on a simplified solution of the scattering problem for an evaporating admixture. Evaporation of water as it is atomized in a turbulent air jet is experimentally studied. Approximate agreement is obtained between the results of the calculations and experiments.

In contrast to evaporation processes of an individual drop, which have been widely studied and are amenable to calculation, actual evaporation processes of systems of drops have been hardly studied at all.

The concept of two evaporation regimes of drop systems in a turbulent gas jet, namely, kinetic and diffusion, has been introduced [1]. The rate of evaporation of the system is determined in the kinetic regime by the kinetic evaporation of an individual drop, and by the rate of diffusion of the external gas as a whole in the diffusion regime. The determination of the evaporation regime in a turbulent drowned jet was carried out by means of the E criterion [1].

Kinetic drop evaporation conditions are realized when $E \gg 1$ and diffusion conditions, when $E \ll 1$.

Drop evaporation in a turbulent drowned jet in the kinetic regime has been considered [2]. It was shown that irreversible ejection of drops from the jet core in the slowly moving periphery at which the evaporation process is consummated is characteristic for the scattering of an evaporating impurity in a turbulent jet. As a result, scattering of the evaporating impurity occurs more rapidly than of the nonevaporating (conservative)

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